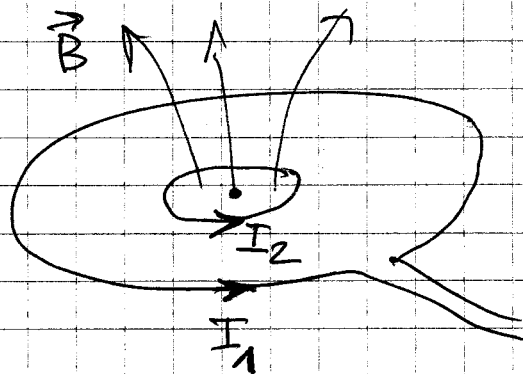


FINAL EXAM SOLUTIONS

- ① The magnetic field \vec{B} in the center



$$B_c = \frac{\mu_0 I_1}{2r_1} \cdot \text{☺}$$

It is time-dependent due to $I_1(t)$.
The magnetic flux through smaller loop is

$$\Phi_2(t) = B_c(t) \cdot A_2 \quad \leftarrow \text{area of the 2nd loop}$$

The magnitude of e.m.f. induced in the smaller loop

$$\begin{aligned} |\mathcal{E}_2| &= \frac{\partial \Phi_2}{\partial t} = \frac{\partial}{\partial t} B_c(t) A_2 = \\ &= A_2 \frac{\mu_0}{2r_1} \frac{dI_1(t)}{dt} = \frac{\mu_0 \pi r_2^2}{2r_1} \frac{dI_1}{dt} \end{aligned}$$

Since $I_1(t) = at$ ($0 \leq t \leq T$)

$$|\mathcal{E}_2| = \frac{\mu_0 \pi r_2^2}{2r_1} \cdot a = \frac{1.26 \cdot 10^{-6} \cdot \pi \cdot 0.1^2}{2 \cdot 1} \times 10$$

$$= 2.0 \cdot 10^{-7} \text{ V}$$

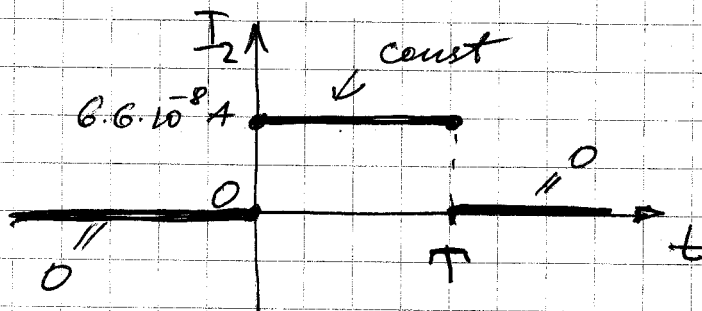
unit check

$$\left[\frac{\text{V} \cdot \text{s}}{\text{A} \cdot \text{m}} \cdot \frac{\text{m}^2}{\text{m}} \cdot \frac{\text{A}}{\text{s}} \right]$$

$$I_2 = \mathcal{E}_2 / R = \frac{2 \cdot 10^{-7}}{3} = \underline{6.6 \cdot 10^{-8} \text{ A}}$$

Direction of this current is
clockwise (opposite to I_1)*

Time evolution of I_2

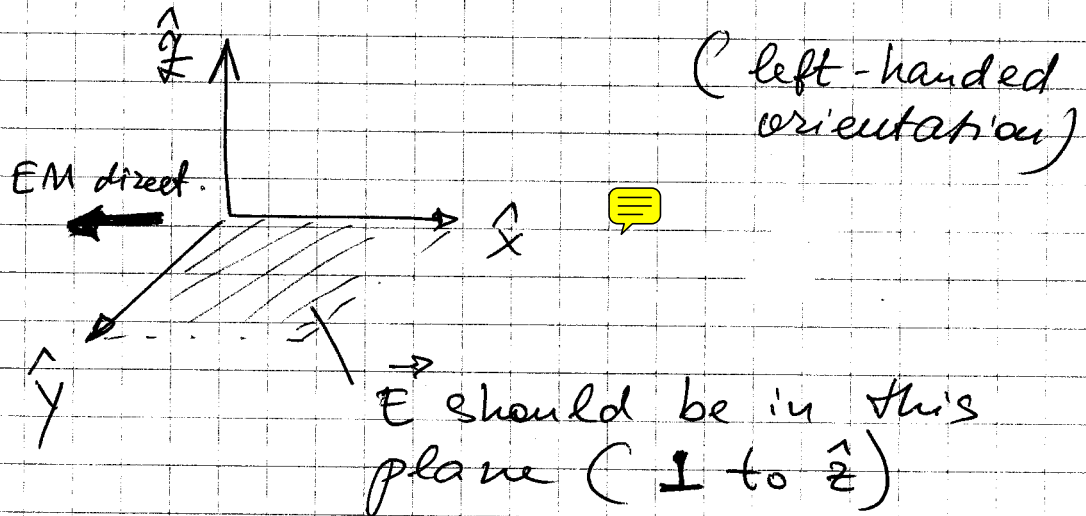


* the magnetic field created by current I_2 induced by the flux Φ_2 will oppose the flux change.

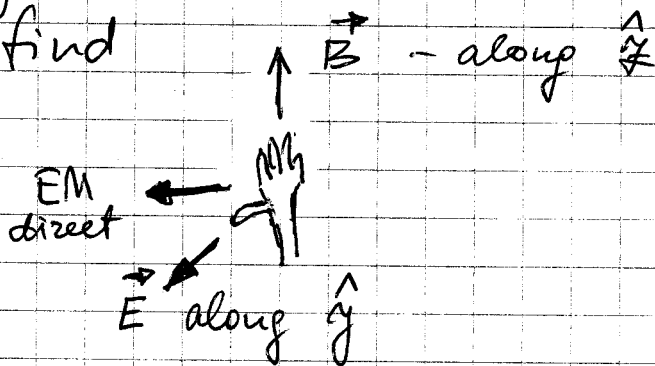
(2)

Direction of EM wave is determined by the cross product

$$\vec{E} \times \vec{B}$$



Using the right hand rule we find



In free space

$$\vec{E}(t) = E_0 \hat{y} \sin \left[\frac{2\pi}{\lambda} (x + ct) \right]$$
$$\vec{B}(t) = B_0 \hat{z} \sin \left[\frac{2\pi}{\lambda} (x + ct) \right]$$

"+" because of $-\hat{x}$ direction

The wave length

$$\lambda = \frac{c}{f} = \frac{3 \cdot 10^8 \text{ m/s}}{100 \cdot 10^6 \text{ s}^{-1}} = \underline{\underline{3 \text{ m}}}$$

Power density $P = 100 \text{ W/m}^2$

The R.M.S. value of the magnetic field that correspond to P is

$$\begin{aligned} B_{\text{RMS}} &= \sqrt{\mu_0 P / c} \\ &= \sqrt{\frac{1.26 \cdot 10^{-6} \cdot 100}{3 \cdot 10^8}} = \sqrt{4.2 \cdot 10^{-13}} \\ &= 6.5 \cdot 10^{-7} \text{ T} \end{aligned}$$

The amplitude of \vec{B} is

$$B_0 = B_{\text{RMS}} \sqrt{2} = \underline{\underline{9.2 \cdot 10^{-7} \text{ T}}}$$

The amplitude of electr. field

$$E_0 = c B_0 = \underline{\underline{274 \text{ V/m}}}$$

③ Kinetic energy [eV] \rightarrow [J]

$$E_k = 10 \cdot 10^3 \text{ eV} = 10 \cdot 10^3 \cdot 1.6 \cdot 10^{-19} \\ = 1.6 \cdot 10^{-15} \text{ J}$$

Initial velocity (neglect relativity)

$$v_0 = \sqrt{2E_k / m_e} = \\ = \sqrt{\frac{2 \cdot 1.6 \cdot 10^{-15}}{9.1 \cdot 10^{-31}}} = \sqrt{3.52 \cdot 10^{15}} = \\ = 5.9 \cdot 10^7 \text{ m/s}$$

Acceleration minus because of slowing down
 $a = -v_0 / t$

t - time required to stop

Stopping distance

$$s = v_0 t + \frac{at^2}{2} = v_0 t - \frac{v_0 t^2}{2t} = \frac{v_0 t}{2}$$

$$t = 2s / v_0 = \frac{2 \cdot 20 \cdot 10^{-10}}{5.9 \cdot 10^7} = 6.7 \cdot 10^{-17} \text{ s}$$

Acceleration $|a| = \frac{5.9 \cdot 10^7}{6.7 \cdot 10^{-17}} = 8.8 \cdot 10^{23} \text{ m/s}^2$

E.M. energy radiated

$$E_{EM} = \frac{t q_e^2 a^2}{6\pi \epsilon_0 c^3} \quad (\text{Larmor formula}) \\ = \frac{6.7 \cdot 10^{-17} \cdot (1.6 \cdot 10^{-19})^2 \cdot (8.8 \cdot 10^{23})^2}{6\pi \cdot 8.9 \cdot 10^{-12} \cdot (3 \cdot 10^8)^3} = \underline{\underline{3.0 \cdot 10^{-22} \text{ J}}}$$

The fraction of energy converted
in E.M. radiation

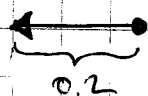
$$\frac{E_{EM}}{E_K} = \frac{3 \cdot 10^{-22}}{1.6 \cdot 10^{-15}} = \underline{\underline{1.9 \cdot 10^{-7}}}$$

- ④ First we calculate reflected light, and then find the amount of photons transmitted

Interface 1 There is a phase flip
(low n to high n
interface)

Let's assume that the light entering the glass has a phase "0"

after " π " - flip



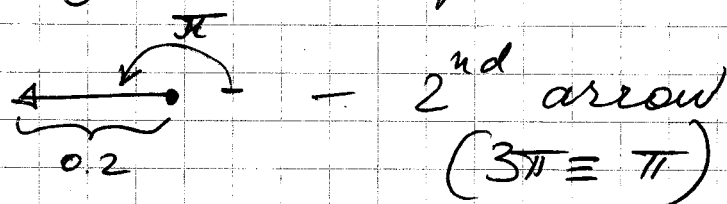
— This represents reflection at point 1

Interface 2 The photon moves through the glass, bounces off the bottom of the glass surface (no phase change here, since it's high n to low n interface) and moves ^{back} through the glass to the surface



The path traveled is $2 \times$ glass thickness = 600 nm , which corresponds to $\frac{600 \text{ nm}}{400 \text{ nm}} = 1.5$

full rotations of the arrow,
 or $1.5 \cdot 2\pi = 3\pi$ phase shift
 with respect to the photons that enters the glass at point "1".



Interface 3

The photon travels

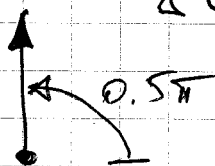
$$2 \cdot (300 + 450) = 2.750 = 1500 \text{ nm}$$

The phase shift $\frac{1500 \text{ nm}}{400 \text{ nm}} \cdot 2\pi = 7.5\pi$
 $\equiv 1.5\pi$

There will be also " π " shift due to (low $n \rightarrow$ high n) interface.

The final phase shift

$$\Delta\phi = 1.5\pi + \pi = 2.5\pi \equiv 0.5\pi$$



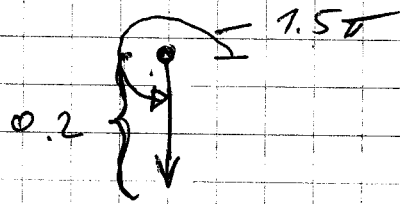
Interface 4

The photon travels

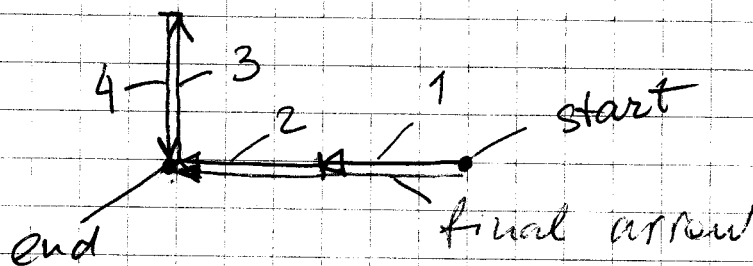
$$2 \cdot (300 + 450 + 200) = 1900 \text{ nm}$$

Phase $\frac{1900 \text{ nm}}{400 \text{ nm}} 2\pi = 9.5 \pi \equiv 1.5\pi$

(no addition phase shift due to high $n \rightarrow$ low n interface)



The final diagram:



Your diagram can be different depending on the choice of phase for the 1st arrow

$$\text{Final arrow length} = 0.2 + 0.2 = 0.4$$

$$\text{The probability } (0.4)^2 = 0.16$$

So 16% of light is reflected, therefore 100 - 16 = 84% is transmitted