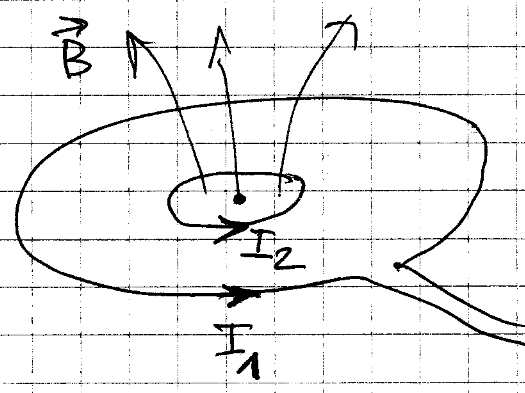
Updated Apr 30, 2013

FINAL EXAM SOLUTIONS

(1) The magnetic field $\vec{B}$ in the center



$$B_c = \frac{\mu_0 I_1}{2 r_1}.$$

It is time-dependent due to $I_1(t)$.

The magnetic flux through smaller loop is

$$\Phi_2(t) = B_c(t) \cdot A_2$$

area of the 2^{nd} loop

The magnitude of e.m.f. induced in the smaller loop

$$|\mathcal{E}_2| = \frac{\partial \Phi_2}{\partial t} = \frac{\partial}{\partial t} B_c(t) A_2 =$$

$$= A_2 \frac{\mu_0}{2 r_1} \frac{d I_1(t)}{dt} = \frac{\mu_0 \pi r_2^2}{2 r_1} \frac{d I_1}{dt}$$

Since $I_1(t) = at \quad (0 \le t \le T)$

$$|\mathcal{E}_2| = \frac{\mu_0 \pi r_2^2}{2 r_1} \cdot a = \frac{1.26 \cdot 10^{-6} \cdot \pi \cdot 0.1^2}{2 \cdot 1} \times 10$$

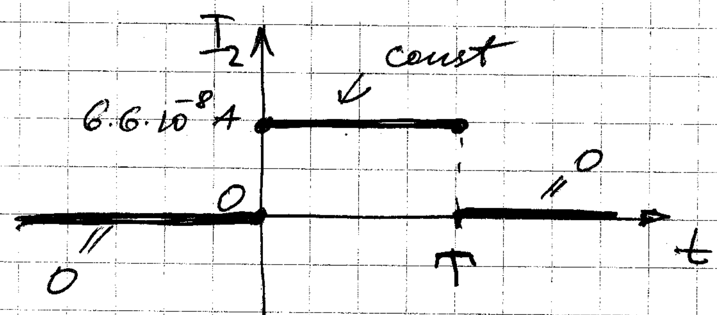
$$= \underline{2.0 \cdot 10^{-7} \text{ V}}$$

unit check $\left[\frac{\frac{V \cdot \cancel{s}}{\cancel{A} \cdot \cancel{m}} \cdot m^{\cancel{2}} \cdot \frac{\cancel{A}}{\cancel{s}}}{\cancel{m}} \right]$

$$I_2 = \mathcal{E}_2 / R = \frac{2 \cdot 10^{-7}}{3} = \underline{6.6 \cdot 10^{-8} \, A}$$

Direction of this current is
<u>clockwise</u> (opposite to I_1)*

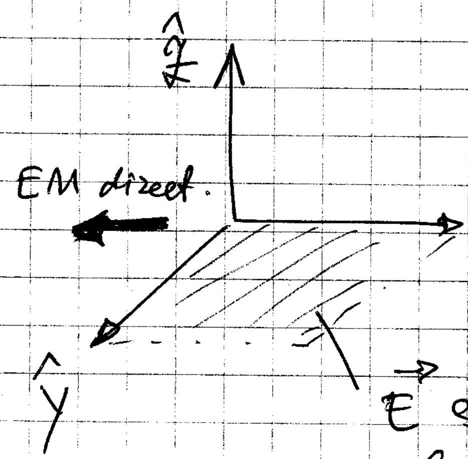
<u>Time evolution</u> of I_2



* the magnetic field created by current I_2 induced by the flux Φ_2 will <u>oppose</u> the flux change.

(2) Direction of EM wave is determined by the cross product

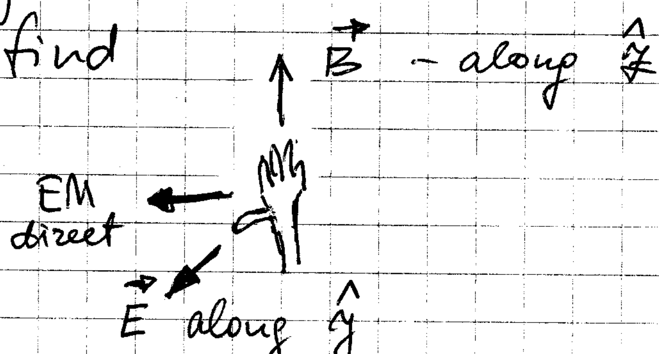
$$\vec{E} \times \vec{B}$$



(left-handed orientation)

$\vec{E}$ should be in this plane ($\perp$ to $\hat{z}$)

Using the right hand rule we find



In free space

"+" because of $-\hat{x}$ direction

$$\boxed{\begin{aligned} \vec{E}(t) &= E_0 \, \hat{y} \, \sin\left[\frac{2\pi}{\lambda}(x + ct)\right] \\ \vec{B}(t) &= B_0 \, \hat{z} \, \sin\left[\frac{2\pi}{\lambda}(x + ct)\right] \end{aligned}}$$

The wave length

$$\lambda = \frac{c}{f} = \frac{3 \cdot 10^8 \ m/s}{100 \cdot 10^6 \ s^{-1}} = \underline{\underline{3 \ m}}$$

Power density $P = 100\ W/m^2$

The <u>R.M.S.</u> value of the magnetic field that correspond to <u>P</u> is

$$B_{RMS} = \sqrt{\mu_0 P / c}$$

$$= \sqrt{\frac{1.26 \cdot 10^{-6} \cdot 100}{3 \cdot 10^{8}}} = \sqrt{4.2 \cdot 10^{-13}} =$$

$$= 6.5 \cdot 10^{-7}\ T$$

The <u>amplitude</u> of $\vec{B}$ is

$$B_0 = B_{RMS} \sqrt{2} = \underline{\underline{9.2 \cdot 10^{-7}\ T}}$$

The amplitude of electr. field

$$E_0 = c B_0 = \underline{\underline{274\ V/m}}$$

(3) Kinetic energy [eV] → [J]

$$E_k = 10 \cdot 10^3\ eV = 10 \cdot 10^3 \cdot 1.6 \cdot 10^{-19}$$
$$= 1.6 \cdot 10^{-15}\ J$$

Initial velocity (neglect relativity)

$$v_0 = \sqrt{2E_k / m_e} =$$
$$= \sqrt{\frac{2 \cdot 1.6 \cdot 10^{-15}}{9.1 \cdot 10^{-31}}} = \sqrt{3.52 \cdot 10^{15}} =$$
$$= 5.9 \cdot 10^7\ m/s$$

Acceleration — minus because of slowing down

$$a = -v_0/t$$

t - time required to stop

Stopping distance

$$S = v_0 t + \frac{at^2}{2} = v_0 t - \frac{v_0 t^2}{2t} = \frac{v_0 t}{2}$$

$$t = 2S/v_0 = \frac{2 \cdot 20 \cdot 10^{-10}}{5.9 \cdot 10^7} = 6.7 \cdot 10^{-17}\ s$$

Acceleration $|a| = \frac{5.9 \cdot 10^7}{6.7 \cdot 10^{-17}} = 8.8 \cdot 10^{23}\ m/s^2$

E.M. energy radiated

$$E_{EM} = \frac{t q_e^2 a^2}{6\pi \varepsilon_0 c^3} \quad \text{(Larmor formula)}$$

$$= \frac{6.7 \cdot 10^{-17} \cdot (1.6 \cdot 10^{-19})^2 \cdot (8.8 \cdot 10^{23})^2}{6\pi \cdot 8.9 \cdot 10^{-12} \cdot (3 \cdot 10^8)^3} = \underline{\underline{3.0 \cdot 10^{-22}\ J}}$$

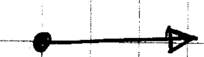
The fraction of energy converted in E.M. radiation

$$\frac{E_{EM}}{E_K} = \frac{3 \cdot 10^{-22}}{1.6 \cdot 10^{-15}} = \underline{\underline{1.9 \cdot 10^{-7}}}$$

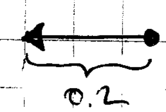
(4) First we calculate reflected light, and then find the amount of photons transmitted

<u>Interface 1</u> There is a phase flip (low <u>n</u> to high <u>n</u> interface)

Let's assume that the light entering the glass has a phase "0"

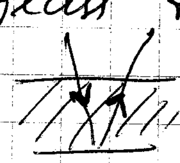


after "π"-flip



— this represents reflection at point <u>1</u>

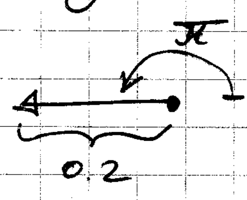
<u>Interface 2</u> The photon moves throug the glas, bounces of the bottom of the glass surface (no phase change here, since it's high <u>n</u> to low <u>n</u> interface) and moves back trough the glass to the surface



The path traveled is $2 \times$ glass thickness $= 600\ nm$, which corresponds to $\frac{600\ nm}{400\ nm} = 1.5$

λ

full rotations of the arrow, or $1.5 \cdot 2\pi = 3\pi$ phase shift with respect to the photons that <u>enters</u> the glass at point "1".



$-\ 2^{nd}$ arrow $(3\pi \equiv \pi)$

<u>Interface 3</u> The photon travels

$$2 \cdot (300 + 450) = 2 \cdot 750 = 1500\ nm$$

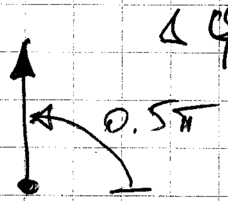
The phase shift $\frac{1500\ nm}{400\ nm} \cdot 2\pi = 7.5\pi$

$$\equiv 1.5\pi$$

There will be also "π" shift due to (low $\underline{n}$ $\rightarrow$ high $\underline{n}$) interface

The final phase shift

$$\Delta\varphi = 1.5\pi + \pi = 2.5\pi \equiv 0.5\pi$$



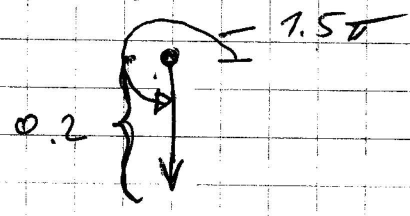
Interface 4

The photon travels

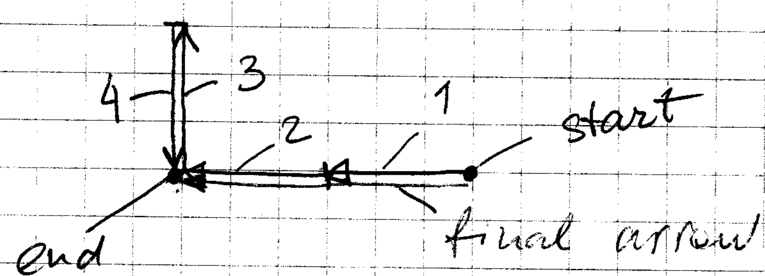
$$2 \cdot (300 + 450 + 200) = 1900 \text{ nm}$$

Phase $\frac{1900 \text{ nm}}{400 \text{ nm}} 2\pi = 9.5\pi \equiv 1.5\pi$

(no addition phase shift due to high n → low n interface)



The final diagram:



Your diagram can be different depending on the choice of phase for the 1st arrow

Final arrow length $= 0.2 + 0.2 =$
$= 0.4$

The probability $(0.4)^2 = 0.16$

So 16% of light is reflected, therefore $100 - 16 = \underline{\underline{84\%}}$ is transmited